by Toby Walsh
September 14, 2009

Global Constraints

Question 1

Given the global constraint $NValues(N, X_1, \ldots, X_m)$, answer the following questions:

1. Consider the reduction of a SAT problem to the solution of a $NValues$ constraint. Construct domains for a $NValues$ constraint so that all solutions of the constraint correspond to models (satisfying assignments) of the following SAT problem: $x_1 \vee x_2$, $\neg x_1 \vee x_2$. Note that the SAT problem has two binary clauses. (5 points)
2. Write down the maximal domains for this problem which are generalized arc-consistent (GAC). (1 point)
3. Explain why all possible values now have support. (2 points)
4. Given the models (satisfying assignments) of the SAT problem and the corresponding supports of the $NValues$ constraint. (2 points)

Question 2

Given a sequence of integer variables, the single peak constraint ensures that values have a single peak. More precisely, it ensures that values increase or stay the same, then decrease or stay the same. For instance, the sequences 011242100 and 112233311 all satisfy the single peak constraint. On the other hand, 123334 does not as the sequence only ever goes up. Similarly 0112113322 does not as there are two peaks (2 and 3). Given a single peak constraint on the domains given below, answer the following questions:

1. Write down the maximal domains which are generalized arc-consistent (GAC). (2 points)
2. Explain why all possible values now have support. (2 points)
3. Assume that variables are integers in the range $[0, 5]$. Specify a deterministic finite automaton which recognizes only sequences of variables satisfying such a single peak constraint (6 points). Note: an automaton is specified by giving the alphabet, the set of states, the unique starting state, the finishing states, and the transition function.

4. Write down a ternary encoding for a single peak constraint on the variables X_1 to X_5 (that is, a sequence of arity 3 constraints). Explain when these ternary constraints are satisfied. (2 points)

5. Simulate by hand enforcing GAC on this encoding with the given domains. Show how every value which is not GAC is pruned. (3 points)

Domains are as follows: $X_1 \in \{2\}$, $X_2 \in \{1, 2, 3\}$, $X_3 \in \{5\}$, $X_4 \in \{1, 2\}$, $X_5 \in \{2, 3\}$.

Question 3

The lex ordering constraint ensures two sequence of vectors are lexicographical ordered. That is, $[X_1] \leq_{\text{lex}} [Y_1]$ iff $X_1 \leq Y_1$. And $[X_1, \ldots, X_n] \leq_{\text{lex}} [Y_1, \ldots, Y_n]$ for $n > 1$ iff ($X_1 \leq Y_1$ and if $X_1 = Y_1$ then $[X_2, \ldots, X_n] \leq_{\text{lex}} [Y_2, \ldots, Y_n]$). Assume that variables are integers in the range $[0, 5]$, and that we have 10 variables, X_1 to X_5 and Y_1 to Y_5.

1. Give an encoding of such a lex ordering constraint using the REGULAR constraint (hint: consider interleaving the sequences). Specify the automaton associated with this REGULAR constraint. (5 points)

2. Write down the maximal domains which are generalized arc-consistent (GAC). (1 point)

3. Explain why all possible values now have support. For each support, give a sequence of states in the deterministic finite automaton which accepts the corresponding string. (4 points)

Domains are: $X_1 \in \{1, 3, 4\}$, $X_2 = 2$, $X_3 \in \{2, 3\}$, $X_4 = 1$, $X_5 \in \{3, 4, 5\}$, $Y_1 = 1$, $Y_2 \in \{0, 1, 2\}$, $Y_3 \in \{2, 3\}$, $Y_4 = 0$, $Y_5 \in \{0, 1, 2\}$.